We develop a general microscopic theory describing the phonon decoherence of quantum dots and indistinguishability of the emitted photons in photonic structures. The coherence is found to depend fundamentally on the dimensionality of the structure resulting in vastly different performance for quantum dots embedded in a nanocavity (0D), waveguide (1D), slab (2D), or bulk medium (3D). In bulk, we find a striking temperature dependence of the dephasing rate scaling as $T^{-1}$ implying that phonons are effectively “frozen out” for $T \lesssim 4$ K. The phonon density of states is strongly modified in 1D and 2D structures leading to a linear temperature scaling for the dephasing strength. The resulting impact on the photon indistinguishability can be important even at sub-Kelvin temperatures. Our findings provide a comprehensive understanding of the fundamental limits to photon indistinguishability in photonic structures.

$$\Gamma_{3D} = \frac{3\pi v_s C_Q^2}{L} \int_0^\infty d(qL)(qL)^{10}e^{-\langle qL \rangle^2}N_q(N_q+1),$$

where $C_Q$ is a dimensionless constant defined later, $L$ the radius of the QD wave function, $v_s$ the speed of sound, and $\langle qL \rangle$ the mean phonon momentum in the longitudinal channel.
The emission spectrum consists of a ZPL and broad sidebands. (b) The quadratic coupling represents scattering of phonons by the QD. In bulk this leads to phonon broadening. (c) The linear coupling is associated with the emission through virtual excitations to a higher state and leads to ZPL (d). (e) Error in two-photon interference versus temperature for QDs embedded in structures with different dimensionality. 0D corresponds to a QD in the center of a sphere with radius $R = 80$ nm, 1D to a cylindrical waveguide with radius $R = 80$ nm and the QD placed in the cross-sectional center (1D), 2D to a QD in the center of a freestanding membrane with height $h = 160$ nm, and 3D to a bulk medium. Each structure is represented by two curves that correspond to a large ($L = 4.5$ nm) and small ($L = 1.5$ nm) wave function denoted with large and small triangles, respectively.

$q$ and $N_q$ the phonon wave number and occupation number, respectively. Remarkably, when the thermal wavelength is larger than the QD size, $\lambda_\text{th} > L$, corresponding to a temperature below a critical temperature $T_c = \hbar v_i/k_B L$, the phonons freeze leading to a rapid drop of the dephasing rate, cf. Fig. 1(d). This yields $\Gamma_{3D}(T < T_c) \approx 3\pi(v_i/L) \times 10! \times C^2_0(T/T_c)^{11}$ leading to highly coherent processes at $T \lesssim 4$ K for realistic QD sizes. Nonostructures on the other hand are finite and can thus expand freely, resulting in long-wavelength vibrations that broaden the ZPL already within the linear exciton-phonon coupling; see Fig. 1(c). The latter competes with the quadratic coupling to yield a nontrivial temperature dependence of the photon indistinguishability; cf. Fig. 1(d). We find that these processes severely limit the coherence in 1D and 2D nanostructures.

To derive the results we generalize the formalism developed in Ref. [14] and consider arbitrary structures with the electron-phonon Hamiltonian

$$H = \hbar \omega_0 |1\rangle\langle 1| + \sum_q \hbar \omega_q a_q^+ a_q + V |1\rangle\langle 1|,$$

where $|1\rangle$ is the QD excited state, $a_q^+$ ($a_q$) the creation (annihilation) operator for the phonon mode with momentum $q$ and energy $\hbar \omega_q$, and $\hbar \omega_0$ is the QD transition energy, cf. Fig. 1(b). We assume low temperatures such that the QD excited states are not populated. The interaction term, $V = V_L + V_Q$, comprises a linear and a quadratic term in phonon displacement

$$V_L = \sum_q L_q A_q, \quad V_Q = \sum_{b,m} \left( \sum_q O_{bq}^m A_q \right)^2,$$

where $A_q = a_q + a_q^+$, $b = \{e, h\}$ denotes electron or hole, $M_{bq}^m$ is the electron-phonon matrix element, and $\Delta_m$ is the energy distance between the ground, $|1\rangle$, and $m$th state of the QD with $m \geq 2$. Time-reversal symmetry implies that all quantities can be chosen real. The interaction with phonons is dominated by the deformation-potential coupling [36,45]

$$M_{bq}^m = D_b \langle \psi^m_b | \nabla \cdot \mathbf{u}_q | \psi^m_b \rangle,$$

where $D_b$ is the deformation-potential constant, $\psi^m_b$ the wave function of the $m$th state, and $\mathbf{u}_q$ the phonon displacement.

After excitation at $t = 0$, the QD coherence is described by the correlation function $P(t) = \langle \sigma^- (t) \sigma^+ (0) \rangle = \langle T e^{-i/\hbar} f_0^{\infty} d\tau \hat{V}(\tau) \rangle$ [14,46], where $T$ is the time-ordering operator, and $\hat{V}$ the potential in the interaction picture with respect to the free phonon Hamiltonian. $P(t)$ can be evaluated numerically exact using the cumulant expansion [14,47], but we follow a simplified approach that captures the essential physics. We find that the QD-phonon interaction is weak, such that retaining the first two terms in the cumulant expansion is sufficient. The distribution, therefore, becomes Gaussian and is completely specified by the mean and standard deviation of the noise $F(t) = -(i/\hbar) \int_0^{\infty} d\tau \hat{V}(\tau)$. The truncation is thus equivalent to treating the phonon bath as a Gaussian noise source, which yields $P(t) \approx \exp \left( -\mu_F t \right) \exp \left( -\frac{1}{2} \left( (T \hat{F}(t) - \mu_F^2) \right) \right)$, where $\mu_F = \langle F(t) \rangle$.
approximation to the exact numerical result. Inserting Eq. (3) into the above expression yields

\[
P(t) = \exp[-i\mu t + K_L(t) + K_Q(t)],
\]

\[
K_L(t) = -\frac{i}{2\hbar} \sum_q |L_q|^2 \int_0^t dt' e^{iD_q(t-t')},
\]

\[
K_Q(t) = \sum_{bmn} \int_0^t dt' \left[ \sum_q Q_{qb}^m Q_{qb}^n D_q(t-t') \right]^2,
\]

where \(D_q(t) = (-i/\hbar)[(N_q + 1)e^{-i\omega_q t} + N_q e^{i\omega_q t}] \) is the phonon Green function. The function \(K_L(t)\) stems from the linear electron-phonon interaction and is determined by a matrix element of the form \(M_{KL}^0(\mathbf{q}) \propto \langle \psi_b^-|\mathbf{\nabla} \cdot \mathbf{u}_q|\psi_b^+\rangle\), which shares the symmetry of the ground-state wave function, implying that \(K_L(t)\) couples to symmetric acoustic deformations, cf. Fig. 1(c). On the other hand, \(K_Q(t)\) is mediated by phonons that share the symmetry of the excited states.

In a bulk medium, the linear interaction \(K_L(t)\) does not contribute to the long-time decay of coherence; see Fig. 2(a). The quadratic coherence function \(P_Q(t) = \exp[K_Q(t)]\) is evaluated for a spherical QD with Gaussian envelopes of radius \(L\). Since \(K_Q(t)\) is proportional to \(\Delta_m^2\), the inclusion of the first triply degenerate excited state, \(m = n = 2\), gives the dominant contribution. Evaluating \(P_Q(t)\) numerically yields a Markovian decay over long time scales, cf. Fig. 2(a), with \(\text{Re}[K_Q(t)] = -\Gamma_{3D}t\) and \(\text{Im}[K_Q(t)]\) contributes to a spectral shift. The dephasing rate can be calculated analytically by performing the time integration in Eq. (5) and using the long-time limit \(\omega_q^{-2}\sin^2\omega_q t \approx \pi t \delta(\omega_q)\). This leads to Eq. (1) with \(C_Q = (D_q^2/\Delta_e + D_h^2/\Delta_h)/3(2\pi)^2\rho_m v_s^2 L^3\). This is plotted in Fig. 2(b) for GaAs parameters: \(v_s = 4780\) ms \(^{-1}\), mass density \(\rho_m = 5.37\) g cm \(^{-3}\), \(D_e = -14.6\) eV, and \(D_h = -4.8\) eV. The energy distance to the excited states is taken to be \(\Delta_e = 2\Delta_h = 40\) meV \(\times L_0/L\) with \(L_0 = 3\) nm, in accordance with theoretical estimates and experimental results [48–54]. This choice of parameters is justified in Ref. [36].

To relate the phonon decoherence to the quality of the photons emitted by the QD, we study a Hong-Ou-Mandel setup [55]. Here, the second-order correlation function determines the two-photon indistinguishability (TPI), which ranges from 0 (no indistinguishability) to 1 (perfect indistinguishability). If (i) the QD-light interaction is Markovian, (ii) the excitation happens instantaneously, (iii) the QD is a perfectly antibunched source of single photons, and (iv) the noise is stationary, the TPI is [56]

\[
TPI = \Gamma_{rad} \int_0^\infty dt e^{-t \Gamma_{rad}} |P(t)|^2.
\]

In bulk, \(|P(t)| \approx \exp(-\Gamma_{3D}t)\) leading to \(TPI = \Gamma_{rad}/(\Gamma_{rad} + 2\Gamma_{3D})\) after filtering out the sidebands, where \(\Gamma_{rad} \approx 2\pi \times 160\) MHz is the radiative decay rate of the QD [57]. The resulting temperature dependence of the TPI is plotted in Fig. 1(d). Near-unity indistinguishabilities can be achieved at temperatures below a few Kelvin. Analytic solutions can also be found at high temperatures, \(\Gamma_{3D}(T > T_c) \approx 3\pi/2 \times (105/32)(v_s/L) \times C_Q^2(T/T_c)^2\), with a quadratic temperature dependence.

In the following we study phonon decoherence in nanostructures [19,20,58,59]. The short-time dynamics results in phonon sidebands that are shaped by the density of states, but this modification is not significant for the sizes considered here [36]. The long-time dynamics can be split into two contributions, \(\tilde{P}_{ZPL}(t) = P_Q(t)P_{L0}(t)\), where \(P_Q(t)\) stems from the quadratic coupling, and \(P_{L0}(t)\) is a nanostructure-specific low-frequency contribution to the linear coupling [21,22]. For simplicity we keep a fixed bulklike radiative decay rate throughout this work. In a realistic device this value may differ, in which case the results should be modified accordingly.

We start with a 0D nanosphere cavity, which resembles the geometry of colloidal QDs embedded in spherical shells [60]. The long-time coherence is plotted in Fig. 3(a) and stems solely from the quadratic coupling \(P_Q(t)\). The decay is strongly non-Markovian because the phonons are reflected at the boundary and interact with the QD periodically; see the inset of Fig. 3(a). A simple expression for \(\text{Re}[K_Q(t)]\) can be derived from Eq. (5) by using the long-time form \(\sin^2(\omega_j - \omega_f) t / (\omega_j - \omega_f)^2 \approx t^2 \delta_{jf}\), where \(j\) is the index of the confined acoustic mode. This results in \(P_{ZPL}(t) = \exp(-S^2 t^2)\) with

\[
S^2 = \frac{3}{2} \left( \frac{\pi v_s}{2L} C_Q \right)^2 \sum_j I_j^{12} \tilde{q}_j^{12} e^{-\tilde{q}_j N_{\tilde{q}_j}(N_{\tilde{q}_j} + 1)},
\]

where \(\tilde{q}_j \equiv q_J L\), and \(I_j\) is a dimensionless normalization factor of the \((j, 1, 0)\) spheroidal mode [36,61]. The resulting
However, in the small temperature limit the energy is smaller than the lowest vibrational state of the phonon modes. We therefore assume the numerically evaluated coherence decay of a QD placed off center at a distance $p/2$ from the center of a cylindrical waveguide of radius $p$ (1D). The decay is non-Markovian, scaling as $P_{1D} = e^{-\Gamma_{1D} t}$ [36]. In both cases, the error in TPI scales as $\propto T$ at low temperatures and is significant even for a waveguide with a diameter of hundreds of nanometers; see Figs. 1(d) and 3(c).

Next we consider a QD embedded in a freestanding 2D membrane with thickness $2h$. To evaluate the coherence, we approximate the dispersion of the fundamental vibrational mode [64] as $\omega = \nu_2 g q l / d$ and discard the modes with $\nu_l > h^{-1}$ [36], where $\nu_2 = \nu_s \sqrt{1 - 2\nu / (1 - \nu)}$, and $q_l$ is the in-plane wave number. The linear scaling of the density of states with $\omega$ yields

$$\text{Re}[K_{2D}] = -p \left[ \gamma_E + \int_0^\infty \frac{\cos \tau}{\tau} d\tau + \ln \tau \right] - \Gamma_{3D} t. \quad (9)$$

Here, $p = (D_e - D_h)^2 (1 - 2\nu)^2 k_B T / 4 \pi \rho_m h v_{1D}^2 (1 - \nu)^2 h^2$, $\Gamma_{3D} = \Gamma_{1D} + \Gamma_{3D}$, is plotted in Fig. 3(a). The coupling to flexural modes, on the other hand, depends on the QD position, and ranges from no coupling at points of high symmetry (e.g., the center of a cylinder) to large coupling away from such points. In Fig. 3(a) we plot the $P_{1D} = e^{-\Gamma_{1D} t}$ [36]. This is different for the linear interaction $P_{1D} = e^{-\Gamma_{1D} t}$. In both cases, the error in TPI scales as $\propto T$ at low temperatures and is significant even for a waveguide with a diameter of hundreds of nanometers; see Figs. 1(d) and 3(c).

FIG. 3. Phonon dephasing in photonic structures and the role of dimensionality. (a) Decay of coherence for the same structures as in Fig. 1(d) except the 0D structure ($R = 20$ nm). Inset: initial decay of coherence (0–20 ps) for bulk (dotted line), sphere (solid line), and the $r^2$ approximation for the sphere. (b) Corresponding emission spectrum. (c) Two-photon indistinguishability versus size of the structure. All plots are for $T = 5$ K, $L = 3$ nm.

emission spectrum without the radiative broadening, $S(\omega) = \text{Re} \int_0^\infty dt P_{2PL}(t) \exp(-i \omega t)$, is Gaussian as depicted in Fig. 3(b). The TPI yields $\text{TPI}_{0D} = \sqrt{\pi} r_s \exp(r_s^2) \text{erfc}(r_s)$, where $r_s = \Gamma_{\text{rad}} / 2 \sqrt{2} S$, and is plotted in Fig. 1(d). In general, the decoherence is stronger than in bulk. However, in the small temperature limit $\lambda_{th} \gg R$, the thermal energy is smaller than the lowest vibrational state of the sphere leading to negligible decoherence, $1 - \text{TPI}_{0D} \propto \exp(-\hbar \omega / 2 k_B T)$, as depicted in Fig. 1(d).

In the following, we discuss the dephasing of QDs embedded in 1D and 2D structures [20,62,63]. The quadratic interaction does not deviate significantly from bulk because $K_Q(t)$ is dominated by phonons with a wavelength comparable to the QD size while realistic photonic structures are much larger and do not affect those phonon modes. We therefore assume $K_{1D} \approx K_{2D} \approx -\Gamma_{3D} t$ [36]. This is different for the linear interaction $P_{1D}$ [22,23]. For a freestanding 1D waveguide two families of acoustic modes contribute to dephasing with a finite $\nabla \cdot \mathbf{u}$: longitudinal expansions of the rod with a linear dispersion and thus a constant density of states at $\omega \to 0$, and flexural modes that bend the rod with a quadratic dispersion and a diverging density of states [36]. The former yield a Markovian decay that was found in Ref. [22] for a cylinder but here is generalized to an arbitrary cross-sectional shape and QD position with the rate

$$\Gamma_{1D} = \frac{(D_e - D_h)^2 (1 - 2\nu)^2 k_B T}{2 A \rho_m v_{1D}^2 \hbar^2}, \quad (8)$$

where $A$ is the cross-sectional area, $\nu = 0.299$ the GaAs Poisson ratio, and $v_{1D} = v_s \sqrt{3 + 2\nu + 2 / (\nu - 1)}$ the phonon speed along the waveguide axis. The total decay, $\exp(-\Gamma_{1D} t)$, with $\Gamma_{1D} = \Gamma_{1D}^{\text{rad}} + \Gamma_{3D}$, is plotted in Fig. 3(a). The coupling to flexural modes, on the other hand, depends on the QD position, and ranges from no coupling at points of high symmetry (e.g., the center of a cylinder) to large coupling away from such points. In Fig. 3(a) we plot the numerically evaluated coherence decay of a QD placed off center at a distance $p/2$ from the center of a cylindrical waveguide of radius $p$ (1D). The decay is non-Markovian, scaling as $P_{1D} = e^{-\Gamma_{1D} t}$ [36]. In both cases, the error in TPI scales as $\propto T$ at low temperatures and is significant even for a waveguide with a diameter of hundreds of nanometers; see Figs. 1(d) and 3(c).

Next we consider a QD embedded in a freestanding 2D membrane with thickness $2h$. To evaluate the coherence, we approximate the dispersion of the fundamental vibrational mode [64] as $\omega = \nu_2 g q || / d$ and discard the modes with $q || > h^{-1}$ [36], where $\nu_2 = \nu_s \sqrt{1 - 2\nu / (1 - \nu)}$, and $q ||$ is the in-plane wave number. The linear scaling of the density of states with $\omega$ yields

$$\text{Re}[K_{2D}] = -p \left[ \gamma_E + \int_0^\infty \frac{\cos \tau}{\tau} d\tau + \ln \tau \right] - \Gamma_{3D} t. \quad (9)$$

Here, $p = (D_e - D_h)^2 (1 - 2\nu)^2 k_B T / 4 \pi \rho_m h v_{1D}^2 (1 - \nu)^2 h^2$, $\Gamma_{3D} = \Gamma_{1D} + \Gamma_{3D}$, is plotted in Fig. 3(a). At long times ($t \gg h / v_{2D}$), Eq. (9) can be simplified to $P_{2D} = (v_{2D} / h)^{-p} \exp(-\Gamma_{3D} t)$. QDs positioned away from the membrane center would also couple to flexural modes with quadratic dispersion resulting in a

FIG. 4. Suppressing phonon dephasing by clamping the photonic structures. The plot shows the photon infidelity versus thickness of SiO$_2$ (gray) surrounding a GaAs waveguide (dark gray) of radius 80 nm with a QD (small black triangle) in the center (solid line) and halfway off center (dashed line). Parameters: $T = 5$ K, $L = 3$ nm, $v_{s, \text{SiO}_2} = 5848$ ms$^{-1}$, $\rho_{m, \text{SiO}_2} = 2.2$ g cm$^{-3}$, $v_{\text{SiO}_2} = 0.17$. 

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Markovian dephasing. Contrary to these examples, the 0D structure has a vanishing density of states at low frequencies. As used above, the dephasing is therefore only due to the quadratic coupling.

The theory developed above directly points towards methods of suppressing the malign impact of phonons. By mechanically clamping the structure, the coupling to the fundamental vibrational mode can be suppressed. This may be achieved by immersing the freestanding structure into another material (e.g., glass or semiconductor [65]) with a lower refractive index such that the light is still guided. We find that thicknesses as small as 1 μm are sufficient to fully suppress the decoherence; see Fig. 4. This may provide a viable approach to obtain near-unity TPI.

In conclusion, we find that the degree of confinement of the nanostructure has a significant impact on the coherence. Bulk (3D) and maximally confined (0D) structures dephase the ZPL solely due to the quadratic exciton-phonon coupling, which becomes negligible at low temperatures, specifically for \( \lambda_{th} \gg L \) and \( \lambda_{th} \gg R \), respectively. The decoherence in 1D and 2D structures is enhanced by long-wavelength vibrations mediated by the linear exciton-phonon coupling and can be important even at sub-Kelvin temperatures. The situation is more involved in the case of more complex structures such as photonic-crystal devices. We expect a photonic-crystal membrane to exhibit worse coherence than a 2D membrane of same thickness due to the holes that would slow down the long-wavelength phonons; see Eq. (9). A detailed calculation of the decoherence in photonic-crystal structures is an interesting question for further investigation.

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Note added.—Recently, a related preprint appeared (and was recently published [66]), which studies the phonon decoherence in bulk systems.

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