Transport of Quantum Noise through Random Media

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We present an experimental study of the propagation of quantum noise in a multiple scattering random medium. Both static and dynamic scattering measurements are performed: the total transmission of noise is related to the mean free path for scattering, while the noise frequency correlation function determines the diffusion constant. The quantum noise observables are found to scale markedly differently with scattering parameters compared to classical noise observables. The measurements are explained with a full quantum model of multiple scattering.

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Light propagation in static disordered photonic media is coherent. The coherence is preserved even after a very large number of scattering events. Coherent transport of light in a disordered medium is the basis for applications of wave scattering to enhance communication capacities [1], for acoustical and biomedical imaging, as well as for fundamental discoveries of intensity correlations, enhanced backscattering, and Anderson localization [2]. All these phenomena are captured by classical physics where, for light, the electromagnetic field is described by Maxwell's wave equation. In contrast, no experiments on multiple scattering have been carried out yet in the realm of quantum optics where a classical description of light is insufficient and effects of the quantized nature of the electromagnetic field are encountered. Pioneering theoretical work includes a study of the propagation of coherent [3] and squeezed light [4] through a random medium with gain, and the associated quantum noise limited information capacities in such random lasers [5]. The photon statistics of a random laser were recently measured [6], which confirmed the expectations for a laser, and it is an instructive example of the independent information that can be extracted by quantum optical measures. Here we present measurements of the transport of quantum and classical noise through a passive multiple scattering medium.

Noise is inevitable in all measurements. The fundamental lower limit is determined by quantum mechanics through Heisenberg's uncertainty relation, and is referred to as the shot noise limit. In the particle description of quantum mechanics, the existence of optical shot noise directly proves that light is quantized [7]. Shot noise fluctuations scale proportional to the square root of the average number of photons (particlelike behavior) in contrast to classical fluctuations that scale linearly (wavelike behavior). The different scaling allows us to distinguish quantum noise from classical noise in an optical experiment. Shot noise is universal to systems consisting of quantized entities and, e.g., offers independent information about the conduction of electrons in mesoscopic conductors compared to standard conductance measurements [8].

In the current Letter we investigate the propagation of classical and quantum intensity noise of light through a multiple scattering, randomly ordered medium. Two different measurements are presented in the noise: total transmission and (short range) frequency correlations. The two measurements provide insight in the static and dynamic transport of quantum noise in a random medium, respectively.

Multiple scattering of light forms a volume intensity speckle pattern of bright and dark spots that most conveniently can be described as a discrete number of conduction channels. We consider the intensity transmitted from an input channel a to an output channel b, $I_\omega^{ab}(t)$, which depends on time t and the optical frequency ω. The intensity is expanded as a mean value $\bar{I}_\omega^{ab}$ plus a fluctuating part $\delta I_\omega^{ab}(t)$ that describes the quantum noise. In a total transmission measurement we sum up all output channels, and the total transmitted intensity is $I_\omega^T(t) = \sum_b I_\omega^{ab}(t)$. The noise transmission coefficient is defined as

$$\mathcal{T}_a^N(\Omega) = \frac{\overline{|\delta I_\omega^T(\Omega)|^2}}{\overline{|\delta I_\omega^{in}(\Omega)|^2}} = \frac{\overline{|\sum_b \delta I_\omega^{ab}(\Omega)|^2}}{\overline{|\delta I_\omega^{in}(\Omega)|^2}}, \tag{1}$$

where the bars denote average over measurement time and Ω is the frequency (Fourier transform of t) that accounts for slowly varying intensity fluctuations of light. $\overline{|\delta I_\omega^{in}(\Omega)|^2}$ is the spectral density of the input noise of the light illuminating the sample through channel a. For a fixed output channel b, we furthermore define the noise autocorrelation function for a frequency offset $\Delta\omega$:

$$\langle\langle C_{ab}^N(\Delta\omega, \Omega)\rangle\rangle_\omega = \frac{\langle\langle \overline{|\delta I_\omega^{ab}(\Omega)|^2} \times \overline{|\delta I_{\omega+\Delta\omega}^{ab}(\Omega)|^2}\rangle\rangle_\omega - \langle\langle \overline{|\delta I_\omega^{ab}(\Omega)|^2}\rangle\rangle_\omega^2}{\langle\langle \overline{|\delta I_\omega^{ab}(\Omega)|^2}\rangle\rangle_\omega^2}, \tag{2}$$

where double brackets $\langle\langle \ldots \rangle\rangle_\omega$ denote an ensemble average that in this case is obtained by averaging over the optical frequency ω. Such noise correlation functions are introduced here for the first time, while substantial effort has centered on intensity correlation functions [9].

The noise spectral density of the transmitted light can be calculated using a full quantum model for multiple scattering [3]. We relate the annihilation operator of the output electric field in channel b ($\hat{a}_\omega^{ab}$) to the input electric field in channel a ($\hat{a}_\omega^{a}$) through the relation

$$\alpha_\omega^{ab} + \hat{a}_\omega^{ab}(t) = t_\omega^{ab}[\alpha_\omega^a + \hat{a}_\omega^a(t)] + \sum_{a' \neq a} t_\omega^{a'b} \hat{a}_\omega^{a'}(t) + \sum_{b'} r_\omega^{b'b} \hat{a}_\omega^{b'}(t), \tag{3}$$

where indices a' and b' label channels on the input and output side of the multiple scattering medium, respectively. $\hat{a}_\omega^{a'}(t)$ and $\hat{a}_\omega^{b'}(t)$ account for vacuum fluctuations in all open channels while $t_\omega^{a'b}$ and $r_\omega^{b'b}$ are electric field transmission and reflection coefficients. In Eq. (3), we have specified the coherent amplitudes of the input field (α_ω^a) and the output field (α_ω^{ab}), and for a coherent state the remaining fluctuations equal vacuum fluctuations, i.e. $\langle \hat{a}_\omega^a(t)\rangle = \langle \hat{a}_\omega^b(t)\rangle = \langle \hat{a}_\omega^{ab}(t)\rangle = 0$ for all a and b. Since we are concerned with intensity fluctuations at frequencies Ω that are very slow compared to the characteristic frequency for transport and change of phase through the scattering sample ($D/L^2 \sim 10^{12}$ Hz) [10] as well as the optical frequency ($\omega \sim 10^{15}$ Hz), it is an excellent approximation to employ a single-longitudinal frequency (ω) for the optical field. Fourier transforming Eq. (3) we calculate the spectral density of the intensity fluctuations $\overline{|\delta I(\Omega)|^2} = \langle(\delta \hat{I}(\Omega))^2\rangle$, where $\hat{I} = \hat{a}^\dagger \hat{a}$ and $\delta\hat{I}(\Omega) = [\alpha^* + \hat{a}^\dagger(\Omega)] \times [\alpha + \hat{a}(\Omega)] - |\alpha|^2$ is a self-adjoint operator. The spectral density is the quantity measured in the experiment, and for a single output channel b we obtain [4]

$$\overline{|\delta I_\omega^{ab}(\Omega)|^2} = |t_\omega^{ab}|^4(\overline{|\delta I_\omega^{in}(\Omega)|^2} - \overline{|\delta I_\omega^{v}(\Omega)|^2}) + |t_\omega^{ab}|^2 \overline{|\delta I_\omega^{v}(\Omega)|^2}, \tag{4}$$

where we have defined the vacuum contribution

$$\overline{|\delta I_\omega^{v}(\Omega)|^2} \equiv \bar{I}_\omega^{in} \langle \hat{a}_\omega^{b'}(\Omega)(\hat{a}_\omega^{b'}(\Omega))^\dagger\rangle, \tag{5}$$

which results from beating between the input field in channel a and vacuum fluctuations from each of the vacuum channels $a' \neq a$ and b' [7]. We note that $\langle \hat{a}_\omega^{b'}(\Omega) \times [\hat{a}_\omega^{b'}(\Omega)]^\dagger\rangle = 1$. Classical noise [in the following referred to as technical noise (TN)] can also be described with Eq. (4) by neglecting all vacuum contributions. In the case of shot noise (SN), we have $\overline{|\delta I_\omega^{v}(\Omega)|^2} = \overline{|\delta I_\omega^{in}(\Omega)|^2}$, and consequently

$$\mathcal{T}_{ab}^{\mathrm{SN}} = \frac{\overline{|\delta I_\omega^{ab}(\Omega)|^2_{\mathrm{SN}}}}{\overline{|\delta I_\omega^{in}(\Omega)|^2}} = T_\omega^{ab}, \tag{6a}$$

$$\mathcal{T}_{ab}^{\mathrm{TN}} = \frac{\overline{|\delta I_\omega^{ab}(\Omega)|^2_{\mathrm{TN}}}}{\overline{|\delta I_\omega^{in}(\Omega)|^2}} = T_\omega^{ab2}, \tag{6b}$$

where $T_\omega^{ab} = |t_\omega^{ab}|^2$ is the intensity transmission coefficient from channel a to b. The noise of the total transmitted intensity can be calculated from Eq. (1). For both shot noise and technical noise it can be shown that the noise of the total intensity is equal to the sum of the noise of each individual channel [11]. After averaging over disorder, we obtain

$$\langle\langle \mathcal{T}_a^{\mathrm{SN}}\rangle\rangle = \frac{\ell}{L}, \tag{7a}$$

$$\langle\langle \mathcal{T}_a^{\mathrm{TN}}\rangle\rangle = \frac{\ell^2}{L^2}, \tag{7b}$$

where ℓ is the transport mean free path for multiple scattering and L the sample thickness. We have omitted contributions from universal conductance fluctuations [11,12] that are negligible for the experiment described in the following. In summary, we have predicted that the total transmissions of quantum and classical noise vary linearly and quadratically with the ratio of the mean free path to the sample thickness, respectively.

Given the transmission coefficients of Eqs. (6), we observe that the noise autocorrelation functions defined in Eq. (2) are either fourth-order or second-order transmission correlation functions for technical noise and shot noise, respectively. Assuming the electric field amplitudes can be described by a circular Gaussian process (short range correlations) [13], it follows that the fourth-order correlation function can be expressed in terms of the second-order correlation function, which implies

$$\langle\langle C_{ab}^{\mathrm{SN}}(\Delta\omega)\rangle\rangle_\omega = f(\eta), \tag{8a}$$

$$\langle\langle C_{ab}^{\mathrm{TN}}(\Delta\omega)\rangle\rangle_\omega = f^2(\eta) + 4f(\eta), \tag{8b}$$

where $f(\eta) = \eta/[\cosh(\sqrt{\eta}) - \cos(\sqrt{\eta})]$ is the second-order correlation function given in [14] with $\eta = 2L^2\Delta\omega/D$.

The experimental setup is outlined in Fig. 1. A frequency tunable titanium-sapphire laser was used to probe the random multiple scattering samples. The lasers' amplitude noise spectrum was found to be limited by shot noise above $\sim$1.5 MHz and dominated by technical noise

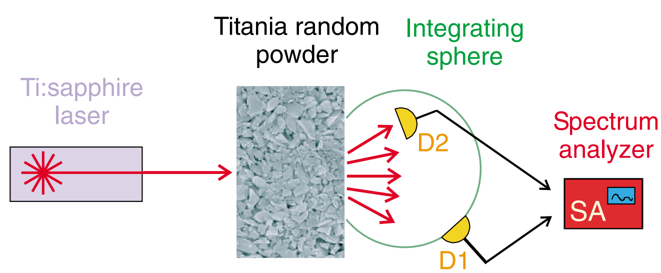

FIG. 1 (color online). Experimental setup for measuring the transmission of quantum noise through a multiple scattering medium. Two different measurements were carried out by inserting either detector D1 or D2. The total transmission was recorded with an integrating sphere onto detector D1. With detector D2, the noise in a single speckle spot was measured.

at lower frequencies, which was carefully checked by observing that the noise of the input beam scaled quadratically and linearly with intensity, respectively. The two regimes enable us to study simultaneously the transmission of classical and quantum noise. We used strongly scattering samples consisting of titania particles (refractive index 2.7) with size distribution $d = 220 \pm 70$ nm, deposited on a fused silica substrate. Two types of experiments were carried out: total transmission and speckle frequency correlation measurements. In the former experiment, the transmitted diffuse light was collected with an integrating sphere onto a sensitive silicon photodiode (detector D1 in Fig. 1). The intensity noise was recorded by measuring the spectral density of the photocurrent $|\delta i(\Omega)|^2$ with a spectrum analyzer. Thermal noise from the detector was subtracted in the measurements. The total transmission of noise was obtained by dividing the measurements with a noise spectrum recorded without any sample inserted. In the speckle correlation measurements we recorded the intensity noise in a single speckle spot (detector D2 in Fig. 1) that was selected using a pinhole. We subsequently varied the frequency of the laser and recorded a frequency speckle pattern. In total, 200 noise spectra were measured at equally spaced optical frequencies with a frequency step of about 0.5 THz. From the complete measurement series the autocorrelation function was obtained.

Figure 2 displays two measurements of the total transmission of noise through samples with different thicknesses. Two frequency regimes are apparent in the data: below ~1 MHz and above ~1.5 MHz, corresponding to the frequencies where the input laser light was dominated by technical noise and shot noise, respectively. We observe immediately that the total transmission of classical noise is significantly lower than the total transmission of quantum noise. Within each noise regime we average over the detection frequency Ω, and ensemble averaging is performed by measuring the transmission several times at different positions on the sample.

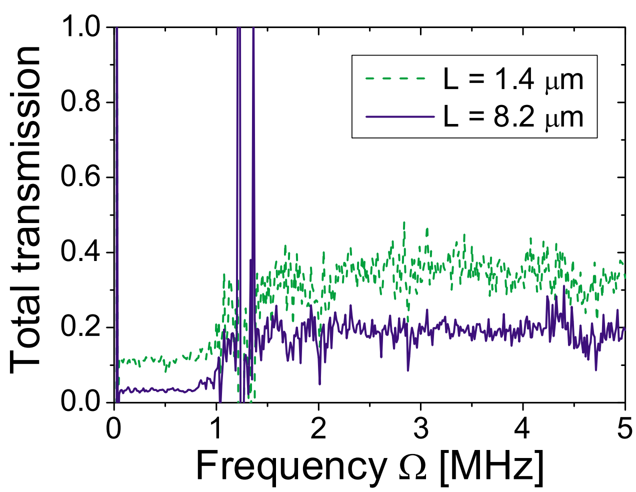

FIG. 2 (color online). Total transmission of noise as a function of measurement frequency Ω for two different sample thicknesses. The spectral densities were recorded with a resolution bandwidth of 30 kHz and a video bandwidth of 10 kHz and by averaging each trace 100 times. Radically different transmissions are observed for technical noise (below 1 MHz) compared to shot noise (above 1.5 MHz). The spikes around 1.3 MHz are due to oscillations in the detector power supply and are abandoned in the analysis.

Figure 3 shows the measured inverse total transmission as a function of sample thickness for both shot noise and technical noise. The experimental data are modeled with the theory in Eqs. (7), and very good agreement is observed. From the fits we extract the transport mean free path, and derive $\ell = 1.19 \pm 0.33$ μm from the shot noise measurements and $\ell = 1.03 \pm 0.09$ μm from the technical noise measurements. The two values agree to within the error bars of the measurements. A proper account for the boundaries of the sample has been included by effectively extending the sample thickness with extrapolation lengths determined by Fresnel corrections [15]. The boundary contributions are determined from the theoretical model at $L = 0$, and full consistency between the two data sets was obtained.

In the speckle correlation measurements we again compare the behavior of classical and quantum noise. The inset in Fig. 4 displays the frequency speckle obtained for shot noise by varying the optical frequency of the incident light. We compute the autocorrelation functions defined in Eq. (2) for both technical noise and shot noise, and their decay with frequency is shown in Fig. 4. The theoretical correlation functions were corrected for a reduced contrast due to stray intensity associated with the selection of a single speckle spot in the experiment. The contrast can be well described by a constant background for both data sets [16]. The correlation function for shot noise is found to

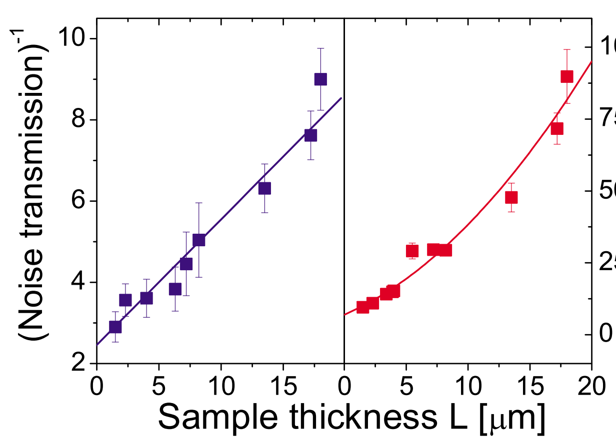

FIG. 3 (color online). Left panel: inverse total transmission of quantum noise as a function of sample thickness. The line is a linear fit to the experimental data. Right panel: inverse total transmission of classical noise and a quadratic fit to the data. The different scales in the two plots clearly demonstrate that classical and quantum noise are transmitted differently.

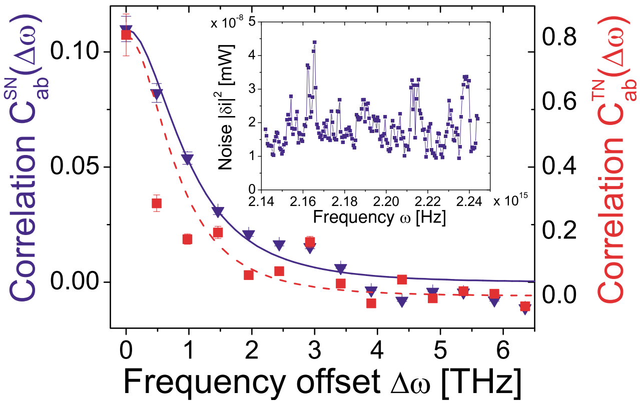

FIG. 4 (color online). Measured correlation function for shot noise (triangles) and technical noise (squares) as a function of frequency offset. The experimental data are compared to $\langle\langle C_{ab}^{\mathrm{SN}}\rangle\rangle$ (full curve) and $\langle\langle C_{ab}^{\mathrm{TN}}\rangle\rangle$ (dashed curve), respectively. Note the different magnitude of the correlation for quantum noise (left axis) and classical noise (right axis). The inset shows the complete shot noise data set of the photocurrent spectral density $|\delta i|^2$ in a speckle spot. Each data point was obtained by averaging the noise spectra over the measurement frequency Ω within the limits of the shot noise region.

extend further in frequency than the correlation function for technical noise, which occurs since the former decays as a second-order correlation function and the latter as a fourth-order correlation function. The measured noise correlation functions can be fitted well by the theoretical prediction of Eqs. (8) using the known value of the sample thickness ($L = 18\ \mu$m). We derive the diffusion constant $D = 34 \pm 2\ \mathrm{m}^2/\mathrm{s}$ from the shot noise data and $D = 30 \pm 4\ \mathrm{m}^2/\mathrm{s}$ from the technical noise data that are both consistent with time-resolved propagation experiments on similar samples. The slight oscillations in the experimental data (most clearly visible for the technical noise data) could be a result of the limited statistics of 200 measurement points corresponding to about 20 independent speckle spots. The good agreement between theory and experimental data for the quantum noise measurements confirms the validity of the quantum model for multiple scattering.

As a side result our experiments demonstrate the robustness of shot noise in multiple scattering: no excess noise was observed due to scattering as opposed to what is expected for an amplifying medium [3,4]. This illustrates an important difference between shot noise in electronics and optics. In a disordered metal wire electronic shot noise is corrupted by thermal noise on the scale of the electron-phonon scattering length [8], and even for purely elastic electron scattering shot noise is reduced by a factor of 3 [17]. On the contrary, optical shot noise prevails over distances much longer than the (elastic) scattering length.

We have studied the propagation of classical and quantum noise through a multiple scattering medium. Both static and dynamic measurements were carried out and compared to theory, which allowed for extracting fundamental scattering properties of the medium. The quantum fluctuations were found to scale markedly differently with scattering parameters compared to classical fluctuations, hence, explicitly demonstrating the difference between particlelike and wavelike transmission.

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